

Resistive Ballooning Modes in Line-Tied Coronal Arcades

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INTRODUCTION

Observations suggest that large scale instabilities in the solar corona, such as solar flares, act to release energy contained in complex magnetic structures, relaxing the fields to a simpler topology. This is possible only if resistive effects play an important role during the flaring process. When considering perturbations of a static equilibrium in a highly conducting magnetized plasma resistivity is usually negligible, as can be seen by examining the linearized induction equation

$$\partial \underline{B}_1 / \partial t = \nabla \times (\underline{v}_1 \times \underline{B}_0) + \eta \nabla^2 \underline{B}_1 \quad (1)$$

unless perturbations are constant along a field line ($\nabla \times (\underline{v}_1 \times \underline{B}_0) = 0$) or the length scale for diffusion becomes small. The presence of the sun's extremely dense photosphere, which anchors magnetic footpoints so that coronal disturbances must vanish there, would seem to exclude the first possibility (Hood 1984) except for very special equilibria (Mok and Van Hoven 1982). On the other hand modes which have a short wavelength perpendicular to the magnetic field and for which the second case occurs, called resistive ballooning modes, are known to be unstable in a wide range of conditions relevant to fusion plasmas (see, e.g. Drake and Antonson 1985). We find that the same is true for arcades in the solar corona.

MODE EQUATIONS

The equations describing the linear evolution of resistive ballooning modes are obtained by using a modified WKB expansion in the short perpendicular wavelength ϵ , while variations of the perturbations along the field are described by a slowly varying amplitude, on which the line tying boundary conditions are imposed. In this way the resistive MHD equations are reduced (to lowest order in ϵ), to a fourth order system of ordinary differential equations for the amplitudes along the field lines:

$$\frac{d\Phi}{d\theta} + r \left[1 + \frac{K^2}{Rm\gamma} \right] / K^2 \frac{\underline{B}^2}{\underline{B}_\theta^2} \underline{A} = 0 \quad (2)$$

$$\frac{d\underline{U}}{d\theta} + \frac{\underline{B}}{\underline{B}_\theta} \left[\frac{\underline{B}^2 + \mu\gamma_s p}{\mu\gamma_s \underline{B}^2} \right] \left[\underline{p}_r + \Phi \frac{d\ln p}{d\ln r} \right] + 2\Phi \frac{\underline{B}_\theta}{\underline{B}} + \frac{K^2}{Rm\gamma} \frac{\mu p}{B\underline{B}_\theta} r \underline{p} = 0 \quad (3)$$

$$\frac{d\underline{p}}{d\theta} + r\gamma^2\mu p \frac{\underline{U}}{p} \frac{\underline{B}}{\underline{B}_\theta} - \frac{\underline{A}}{K^2} \frac{\underline{B}^2}{\underline{B}_\theta^2} \frac{d\ln p}{d\ln r} = 0 \quad (4)$$

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C - 6

$$\frac{dA}{d\theta} - 2\tilde{p} \frac{\mu p}{B^2} + \mu \rho \gamma^2 \tilde{\Phi} \frac{K^2}{B^2} = 0 \quad (5)$$

The arcade is symmetric around $\theta = 0$ and extends infinitely in the z -direction. The photosphere is located at $\theta = -\pi/2$ and $\theta = \pi/2$, where the line tying conditions are $\tilde{\Phi} = 0$, $\tilde{U} = 0$. $\tilde{\Phi}$, \tilde{p} , A , \tilde{U} are the perturbed scalar potential, pressure, parallel vector potential and flow velocity respectively. K^2 is the squared norm or the perpendicular wave number, and is a quadratic function of θ . R_m is the magnetic Reynolds number for the length scale ϵ . Equations (2)-(5) depend parametrically on r , and the resulting growth rate γ also is a function $\gamma = \gamma(r)$. The radial structure of the mode is therefore not established to lowest order. Details of this problem may be found in Hood, 1986. However, the maximum of the function $\gamma(r)$, when it is positive, is the actual physical growth rate of the mode (Connor, Hastie and Taylor 1978). The driving term for the instability is the radial pressure gradient, when it is negative, while magnetic shear and compressibility tend to have a stabilising effect. Our results will be illustrated for the equilibrium

$$\begin{aligned} B_\theta &= B_0 r e^{-r/z} \\ B_z &= \lambda B_0 (\sigma + (2 + 2r - rz)e^{-r/z})^{1/2} \\ \mu p &= (1 - \lambda^2) B_0^2 / 2 (\sigma + (2 + 2r - r^2)e^{-r/z}) \end{aligned} \quad (6)$$

which depends on the parameters λ , σ . The same results hold in general because the local analysis depends on the values of fields and pressure on each surface independently.

RESULTS

In general, given an equilibrium, there are certain ranges of magnetic surfaces for which the system (2)-(15) predicts instability even without dissipation (Hood 1986). As expected we find that in this case resistivity has little influence on the growth rates that are found. On the other hand, in regions where the equilibrium is stable to ideal modes, we find that resistivity introduces a purely growing mode with eigenvalue γ depending linearly on the inverse magnetic Reynolds number R_m^{-1} . As ideal marginal stability is approached, or alternatively if the perpendicular wavelength is decreased, one finds that the power dependence decreases gradually to $\gamma \propto R_m^{-1/3}$, as shown in Fig. 1, where curves for different values of the equilibrium parameters are shown. The main conclusion is that within the resistive MHD approximation cylindrically symmetric arcades with pressure falling with radius are unstable to resistive localised modes; the growth rates, close to ideal marginal stability, are large, typically in the range $10^{-2} \omega_A < \gamma < 10^{-1} \omega_A$ so that it would appear that energy could be released during 10–100 Alfvén times. The wavelength of the modes is expected to be limited by the ion gyroradius, when stabilising drift effects must be taken into account. On the other hand the suggestion has been made (Weiland and Mondt 1985) that the nonlinear development of these localised modes could lead to an explosive instability. In any case the nonlinear evolution of resistive ballooning modes should be studied to assess their overall relevance to the violent and rapidly evolving phenomena observed on the sun.

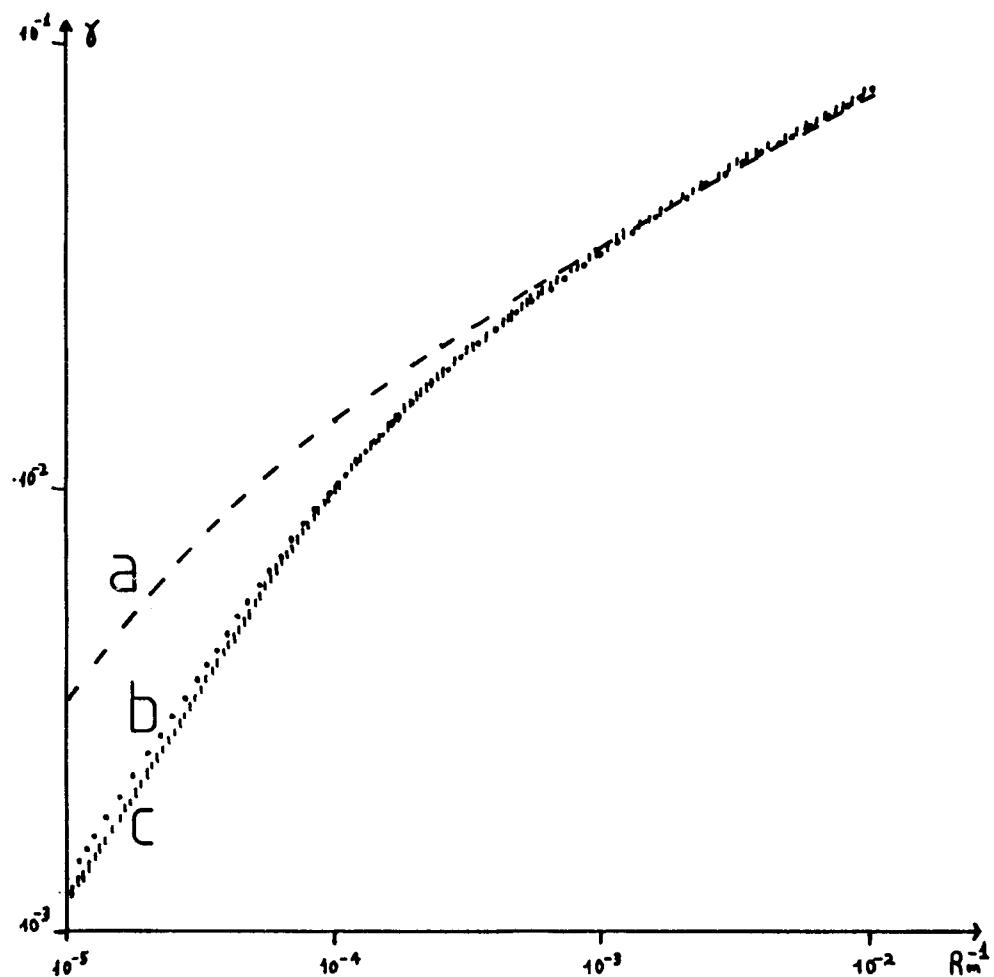


Fig. 1. Growth rate γ (normalised to the Alfvén frequency) as a function of R_m^{-1} for different values of equilibrium parameters: a- $\lambda = 0.14$, $\sigma = 0.25$, $r = 1.5$, b- $\lambda = 0.2$, $\sigma = 0.15$, $r = 1.76$, c- $\lambda = 0.21$, $\sigma = 0.20$, $r = 1.65$. In all cases, the equilibrium is ideally stable at every radius.

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